Analysis of the Y(2175) as a tetraquark state with QCD sum rules

Zhi-Gang Wang [1]

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we take the point of view that the $Y(2175)$ be a tetraquark state which consists of the color octet constituents, and calculate its mass and decay constant within the framework of the QCD sum rule approach. The value of the $Y(2175)$ is consistent with the experimental data, there may be some tetraquark components in the state $Y(2175)$. The tetraquark states may consist of the color octet constituents rather than the diquark pairs.

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1 Introduction

Recently, the Babar collaboration observed a resonance with the quantum numbers $J^{PC} = 1^{--}$ near the threshold in the process $e^+e^- \to \phi f_0(980)$ via initial-state radiation [1]. The Breit-Wigner mass is $m_Y = (2.175 \pm 0.010 \pm 0.015)GeV$ and the width is narrow $\Gamma_Y = (58 \pm 16 \pm 20)MeV$. The resonance may be interpreted as an $\bar{s}s$ analogue of the $Y(4260)$, or as an $\bar{s}s\bar{s}s$ state that decays predominantly to the $\phi f_0(980)$. In this article, we take the point of view that the state $Y(2175)$ (thereafter we take the notation $Y(2175)$) be a tetraquark state with the quantum numbers $J^{PC} = 1^{--}$, and calculate its mass and decay constant in the framework of the QCD sum rules approach [2]. Whether or not there exist a tetraquark configuration $\bar{s}s\bar{s}s$ which can result in the baryonium state is of great importance itself, because it provides a new opportunity for a deeper understanding of the low energy QCD. We explore this possibility, later experimental data can confirm or reject this assumption. The interactions of the one-gluon exchange and direct instantons lead to significant attractions between the quarks in the 0^+ channel, the $Y(4260)$ can be taken as consist of the scalar diquark ($\epsilon_{kij} c_i^T C\gamma_5 s_j$) pairs in relative P-wave [3]. However, two s quarks can not cluster together to form a scalar diquark, if the $Y(2175)$ is the cousin of the $Y(4260)$, why they have so different substructures?

The article is arranged as follows: we derive the QCD sum rules for the mass and decay constant of the $Y(2175)$ in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

[1] E-mail,wangzgyiti@yahoo.com.cn.

2 QCD sum rules for the mass of the $Y(2175)$

In the following, we write down the two-point correlation function $\Pi_{\mu\nu}(p^2)$ in the framework of the QCD sum rules approach,

$$
\begin{aligned}
\Pi_{\mu\nu}(p^2) &= i \int d^4x e^{ip\cdot x} \langle 0|T\{J_\mu(x) J_\nu^+(0)\}|0\rangle \,, && (1)\\
J_\mu(x) &= \bar{s}(x)\gamma_\mu \lambda^a s(x) \bar{s}(x) \lambda^a s(x) \,, && (2)\\
f_Y m_Y^4 \epsilon_\mu &= \langle 0|J_\mu(0)|Y\rangle \,. && (3)
\end{aligned}
$$

Here the λ^a's are the Gell-Mann matrixes for the color $SU(3)$ group, the ϵ_μ and the f_Y are the polarization vector and decay constant of the $Y(2175)$ respectively. We take the color octet operators $\bar{s}(x)\gamma_\mu\lambda^a s(x)$ and $\bar{s}(x)\lambda^a s(x)$ as the basic constituents in constructing the vector current $J_\mu(x)$. Originally, the color octet operators $\bar{s}\lambda^a s$, $\bar{q}\lambda^a q$, $\bar{s}\gamma_5\lambda^a s$ and $\bar{q}\gamma_5\lambda^a q$ were used to construct the interpolating currents for the scalar meson $a_0(980)(f_0(980))$ as a tetraquark state [4]. There are other two vector current operators $J^A_\mu(x)$ and $J^B_\mu(x)$ with the same quantum numbers $J^{PC} = 1^{--}$ as the $Y(2175)$,

$$
\begin{aligned}
J^A_\mu(x) &= \bar{s}(x)\gamma_\mu s(x)\bar{s}(x)s(x) \,, \\
J^B(x) &= \epsilon^{kij}\epsilon^{kmn}\left\{s_i^T(x)C\sigma_{\mu\nu}s_j(x)\bar{s}_m(x)C\gamma^\nu\gamma_5\bar{s}_n^T(x)\right. \\
&\quad \left. +\bar{s}_i(x)C\sigma_{\mu\nu}\bar{s}_j^T(x)s_m^T(x)C\gamma^\nu\gamma_5 s_n(x)\right\} \,. && (4)
\end{aligned}
$$

Here the k, i, j, m, n are the color indexes, the C is the charge conjunction matrix, the μ and ν are the Lorentz indexes. If we take the color singlet operators $\bar{s}(x)\gamma_\mu s(x)$ and $\bar{s}(x)s(x)$ as the basic constituents, and choose the current operator $J^A_\mu(x)$, which can interpolate a tetraquark state, whether the compact state or the loose deuteron-like $\phi f_0(980)$ bound state, it is difficult to separate the contributions of the bound state from the scattering $\phi f_0(980)$ state. In this article, we take the $Y(2175)$ as a baryonium state and choose the current $J_\mu(x)$, although the $J^A_\mu(x)$ has non-vanishing coupling with the $Y(2175)$. In the diquark-antidiquark model [3], the $Y(4260)$ is taken as consist of the scalar diquark ($\epsilon_{kij}c_i^T C\gamma_5 s_j$) pairs in relative P-wave. One can take the $Y(2175)$ as the cousin of the $Y(4260)$, the decays $Y(4260) \to J/\psi f_0(980)$ and $Y(2175) \to \phi f_0(980)$ occur with the same mechanism, however, the $Y(2175)$ can not be constructed from the scalar ss diquark pairs, because two s quarks can not cluster together to form a scalar diquark due to the Fermi statistics, we have to resort to the constituents, a tensor diquark and a vector diquark in relative S-wave, to construct the interpolating current, if one insist on that the multiquark current operators should be constructed from the diquark pairs. It is odd that the cousins have very different substructures, the $Y(4260)$ may have the structure $\bar{c}\gamma_\mu\lambda^a c\bar{s}\lambda^a s + \bar{s}\gamma_\mu\lambda^a s\bar{c}\lambda^a c$.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [2], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator $J_\mu(x)$

into the correlation function in Eq.(1) to obtain the hadronic representation. After isolating the pole term of the lowest $Y(2175)$ state, we obtain the following result,

$$\Pi_{\mu\nu}(p^2) = -\frac{f_Y^2 m_Y^8}{m_Y^2-p^2}\left\{g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right\}+\cdots . \tag{5}$$

We carry out the operator product expansion to the vacuum condensates adding up to dimension-11 to obtain the correlation function $\Pi_{\mu\nu}(p^2)$ at the level of quark-gluon degrees of freedom. In calculation, we take the assumption of the vacuum saturation for the high dimension vacuum condensates, they are always factorized to lower condensates with the vacuum saturation in the QCD sum rules, the factorization works well in the large N_c limit. In this article, we take into account the contributions from the quark condensate $\langle\bar{s}s\rangle$, mixed condensate $\langle\bar{s}g\sigma Gs\rangle$, gluon condensate $\langle\frac{\alpha GG}{\pi}\rangle$, and neglect the contributions from other high dimension condensates, which are suppressed by large denominators and would not play significant roles. Once the analytical results are obtained, then we can take the current-hadron duality below the threshold s_0 and perform the Borel transformation with respect to the variable $P^2=-p^2$, finally we obtain the following sum rule,

$$f_X^2 m_Y^8 e^{-\frac{m_Y^2}{M^2}} = \int_{16m_s^2}^{s_0} dte^{-\frac{t}{M^2}}\frac{\mathrm{Im}\Pi(t)}{\pi}+\frac{\langle\bar{s}g\sigma Gs\rangle^2}{72\pi^2}+\frac{40m_s\langle\bar{s}s\rangle^3}{27}-\frac{16m_s\langle\bar{s}s\rangle^2\langle\bar{s}g\sigma Gs\rangle}{81M^2}, \tag{6}$$

$$\begin{aligned}\frac{\mathrm{Im}\Pi(t)}{\pi} &= \frac{t^4}{27648\pi^6}+\frac{m_s\langle\bar{s}s\rangle t^2}{72\pi^4}-\frac{t^2}{13824\pi^4}\langle\frac{\alpha GG}{\pi}\rangle-\frac{m_s\langle\bar{s}g\sigma Gs\rangle t}{216\pi^4}\\ &\quad+\frac{\langle\bar{s}s\rangle^2 t}{27\pi^2}-\frac{\langle\bar{s}s\rangle\langle\bar{s}g\sigma Gs\rangle}{18\pi^2}.\end{aligned} \tag{7}$$

Differentiate the above sum rule with respect to the variable $\frac{1}{M^2}$, then eliminate the quantity f_Y, we obtain the QCD sum rule for the mass,

$$\begin{aligned}m_Y^2 &= \left\{\int_{16m_s^2}^{s_0} dtte^{-\frac{t}{M^2}}\frac{\mathrm{Im}\Pi(t)}{\pi}+\frac{16m_s\langle\bar{s}s\rangle^2\langle\bar{s}g\sigma Gs\rangle}{81}\right\}/\\ &\quad\left\{\int_{16m_s^2}^{s_0} dte^{-\frac{t}{M^2}}\frac{\mathrm{Im}\Pi(t)}{\pi}+\frac{\langle\bar{s}g\sigma Gs\rangle^2}{72\pi^2}+\frac{40m_s\langle ss\rangle^3}{27}-\frac{16m_s\langle\bar{s}s\rangle^2\langle\bar{s}g\sigma Gs\rangle}{81M^2}\right\}.\end{aligned} \tag{8}$$

It is easy to integrate over the variable t, we prefer this formulation for simplicity.

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle\bar{q}q\rangle=-(0.24\pm0.01GeV)^3$, $\langle\bar{s}s\rangle=(0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{s}g\sigma Gs\rangle=m_0^2\langle\bar{s}s\rangle$, $m_0^2=(0.8\pm0.1)GeV^2$, $\langle\frac{\alpha GG}{\pi}\rangle=$

$(0.33GeV)^4$ and $m_s = (0.14 \pm 0.01)GeV$. For the multiquark states, the contributions from the terms with the gluon condensate $\langle \frac{\alpha GG}{\pi} \rangle$ are of minor importance, and the uncertainty is neglected here [5, 6]. The main contribution to the correlation function in Eq.(6) comes from the perturbative term, the standard criterion of the QCD sum rules can be satisfied; which is in contrast to the ordinary sum rules with the interpolating currents constructed from the multiquark configurations, where the contribution comes from the perturbative term is very small [7], the main contributions come from the terms with the quark condensates $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$, sometimes the mixed condensates $\langle \bar{q}g\sigma Gq \rangle$ and $\langle \bar{s}g\sigma Gs \rangle$ also play important roles [5, 6]. Neglecting the contributions of the vacuum condensates and taking the values $\sqrt{s_0} = 2.4GeV$, $M^2 = (3-7)GeV^2$, we can obtain the value $m_Y = 2.17GeV$, it is indeed the main contribution comes from the perturbative term. If we take the color octet operators $\bar{q}\lambda^a q$, $\bar{q}i\gamma_5\lambda^a q$, $\bar{q}\gamma_\mu\lambda^a q$, $\bar{q}\gamma_\mu\gamma_5\lambda^a q$ and $\bar{q}\sigma_{\mu\nu}\lambda^a q$ as the basic constituents to construct the tetraquark currents, the contributions of the perturbative terms may have the dominant contributions, in other words, the tetraquark states may consist of the color octet constituents rather than the diquark pairs [3].

For the conventional (two-quark) mesons and (three-quark) baryons, the hadronic spectral densities are experimentally well known, the separation between the ground state and excited states is large enough, the "single-pole + continuum states" model works well in representing the phenomenological spectral densities. The continuum states can be approximated by the contributions from the asymptotic quarks and gluons, and the single-pole dominance condition can be well satisfied,

$$\int_{s_0}^{\infty} \rho_A e^{-\frac{s}{M^2}} ds < \int_0^{s_0} (\rho_A + \rho_B) e^{-\frac{s}{M^2}} ds \, , \tag{9}$$

where the ρ_A and ρ_B stand for the contributions from the perturbative and non-perturbative part of the spectral density respectively. However, the present experimental knowledge about the phenomenological hadronic spectral densities of the multiquark states is rather vague, even the existence of the multiquark states is not confirmed with confidence, and no knowledge about either there are high resonances or not. The condition in Eq.(9) can not lead to reasonable Borel parameter M and threshold parameter s_0 for the multiquark states, we can either reject the QCD sum rules for the multiquark states or release the condition, we are optimistical participators. We choose the suitable values of the Borel parameter M, on the one hand, the minimal values M_{min} are large enough to warrant the convergence of the operator product expansion, for $M_{min} > \sqrt{3}GeV$, the dominating contribution comes from the perturbative term, larger than 90%; on the other hand, the maximal values M_{max} are small enough to suppress the contributions from the high excited states and continuum states, we choose the naive analysis $e^{-s_0/(M_{max})^2} \leq e^{-1}$.

We approximate the spectral density with the contribution from the single-pole term, the threshold parameter s_0 is taken slightly above the ground state mass ($\sqrt{s_0} > m_Y + \frac{\Gamma_Y}{2}$) to subtract the contributions from the excited states and continuum states, for detailed discussions about the criteria for selecting the Borel parameter

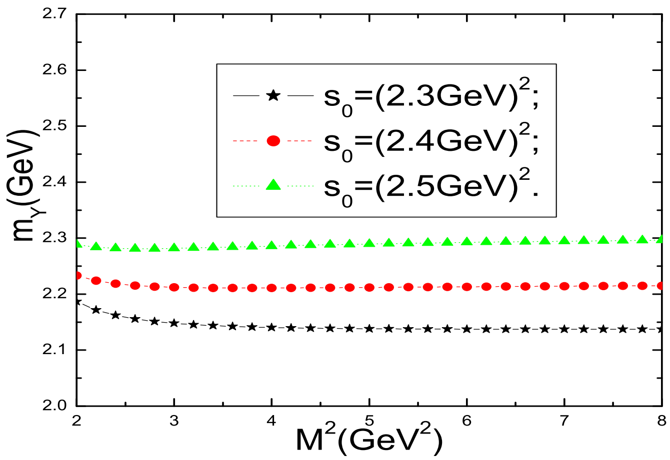

Figure 1: The m_Y with the Borel parameter M^2 for the central values of the condensates.

and the threshold parameter for the multiquark states, one can consult Ref.[6]. In this article, the threshold parameter s_0 is taken to be $\sqrt{s_0} = (2.3-2.5)GeV > 2.2GeV$, it is reasonable for the Breit-Wigner mass $m_Y = 2.175 \pm 0.010 \pm 0.015GeV$ and width $\Gamma_Y = 58 \pm 16 \pm 20MeV$. The Borel parameter M can be chosen to be $M^2 = (3.0-7.0)GeV^2$, in this region, the values of the mass and decay constant are rather stable with respect to variation of the Borel parameter, which are shown in Fig.1 and Fig.2 respectively. Finally, we obtain the values of the mass and the decay constant of the $Y(2175)$,

$$
\begin{aligned}
m_Y &= (2.21 \pm 0.08)GeV\,, \\
f_Y &= (5.78 \pm 0.71)10^{-4}GeV\,.
\end{aligned}
\tag{10}
$$

4 Conclusion

In this article, we take the point of view that the $Y(2175)$ be a tetraquark state which consists of the color octet constituents, and calculate its mass and decay constant within the framework of the QCD sum rule approach. The value of the $Y(2175)$ is consistent with the experimental data, there may be some tetraquark components in the $Y(2175)$ state. We can take the color octet operators as the basic constituents in constructing the tetraquark currents, because the perturbative term may have dominant contribution, in other words, the tetraquark states may consist of the color octet constituents rather than the diquark pairs.

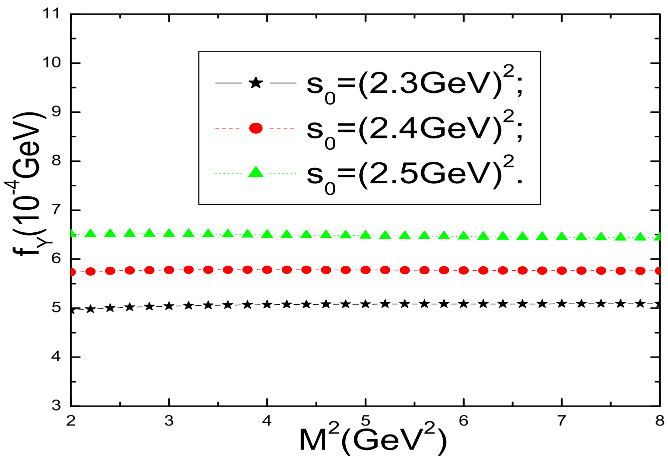

Figure 2: The f_Y with the Borel parameter M^2 for the central values of the condensates.

Acknowledgments

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